## 构建数学模型计算登机时间

### 常规状态

(P16)

In this model, we will calculate the total time of boarding. According to the discreteness of our model, this task can be reduced to finding a recursion formula for any passenger-based variable. Out of simplicity and authenticity concerns, we chose (velocity) as that valuable.

Before turning the spotlight on the analysis, we’ll first construct the space of cells and coordinates as shown on the slide.

(P17)

In addition, so far we are only looking into the regular cases where passengers move freely without being blocked. In this case, there doesn’t exist scenarios of contradictions such as .

(P18)

First, we’ll calculate the velocity according to the density. The density defined in the model is as shown in the slide. The visibility range is taken as because is not too big or too small and is realistic, and it also decides the time step (if taken as , the time step can be or sec), reducing the complexity of simulation.

(P19)

Next, e use Greenshields speed-density linear model (Ref. 3 in essay) to develop the relation between and .

(P20)

After that, we use dot products of vectors to calculate distribution according to density, as the slide shows.

(P21)

Additionally, by using partial summation, the distribution also has correspondent associations with the velocities.

(P22)

Therefore, we get the result.

Notice that previous calculations have shown that real-time speeds are associated under a linear bound. And we’ll also use two methods to justify our deductions: first on the next slide and then in the Sensitivity Analysis part.

(P23)

This is the recursion formula. It clearly displays the linearity. ( can be understood as the real-life speed.)

**Add diagram for gradual deduction**

### 拥挤状态

(P24)

As mentioned before, now we’ll come to the second scenario: when someone is causing a queue.

We divide the task into two parts: stowing luggage and offering seats. The first is trivial (but we’ll later add *discompliance* factors to this in the SA), and the latter can be calculated as shown mathematically – using permutation and the preservation of order.

(P25)

Here are the calculations.

(P26)

Here is the schematic diagram for this procedure.

### 状态的转换

(P27)

Now we’ll show the formula for the interconversion of states. The formulas here are further improved compared to our essay.

(P28)

Here are the ideal formula. It preserves the linearity.

(P29)

Deletion is rather trivial according to programmatic views.

### 结论部分

(P30)

Here we give the results. The weights can be calculated by accumulating all the and select the element. This can be easily be done with matrix multiplications:



(P31)

After obtaining all these indicators, we calculate the total time.

## 最优化

(P32)

In this part, we’ll focus on the modelling approach to optimizing (or minimizing) total time. Our work can be divided into two parts: inspiration from our previous calculations and the strict mathematical proof.

We’ll raise this concept named *parallelity* to describe how many of the aisle cells are occupied. Intuitively, the higher the parallelity, the more efficient the system is, and the faster the strategy is. The formulae of parallelity is shown in the slide.

(P33)

We’ll prove the intuitive idea proposed in the previous slide.

First, based on the model, we can do these analysis as shown.

(P34)

These properties are preserved by the linearity of our model.

(P35)

Secondly and mathematically, we’ll also prove this with two major claims. The first is about the optimality of all cells being occupied. The second will be useful when dealing with more complicated aircrafts.

(P36)

Here is Claim one. You can also refer to this in the essay. As the thesis has been shown before, we’ll not elaborate on this.

**P40**

These are the results of our simulation.

**P41**

N/A.

**P42**

N/A.

**P43**

This is a comparison between different methods. We can see that **Steffen Sub-Perfect performs the best overall** with **Steffen Perfect following**, and back-to-front, windle-to-aisle following; it is evident that the **random method** even outweighs the **front-to-back method**.

**P44**

We need to conduct a sensitivity analysis on our model, but how? We use **compliancy index** to measure in figure. It shows the **predictability of changes** in the model, and the function to compare with is a relationship proved by facts.

**P45**

Case one of our Sensitivity Analysis is a **longer stowing time**. We use a random model – **sigmoid model**, as shown in the slides – to **distribute** the dicompliance of passengers in a relatively realistic method. We choose the sigmoid model due to its speciality in its functions and value, as shown in the figure to the right.

**P46**

These are how we determine whether the model is stable or unstable based on how the graph looks.

**P47**

N/A.

**P48**

We can conclude that random boarding is the most sensitive while front-to-back seems not sensitive.

**P49**

Next, we analysed the queue-jumping situation and concluded that both methods are sensitive, meaning queue-jumping significantly impacts total results.

**P50**

Last but not least, we researched the reduction of passengers and found out that random boarding is the most sensitive (see the distribution of points) while back-to-front is not so sensitive.

**P51**

These are the major conclusions drawn from our sensitivity analysis: Random is far more sensitive than front to back, because randomised sequences can result in immeasurable effects. Back-to-front is the best overall because it is the least sensitive and has better time and satisfaction.

P5

**Our model can be divided into three parts: Math Model, Optimization in a mathematical account and Program.** The program part can verify the other two parts, and the optimization part can verify the models. The Math model part mainly discusses the factors that decide a plan is good or not: total boarding time and passenger satisfaction, and also includes some optimization. The optimization part is about ways to make the strategy better. And the program part is to simulate the boarding process, to verify our conclusion given by the model part.

P6

The inherent structure of the problem indicates that **our model will be discrete.** Therefore, apart from the intuitive ones, other assumptions, such as the third one in this slide, will hinge on this property of discreteness. On the other hand, **it’s also important to make our assumptions plausible.** This is the main reason for making most of the **seemingly impulsive hypothesis** afterwards.

P7

**These two assumptions are about the moving state of passengers and we’ve given correspondent justifications.**

P8

**These assumptions are respectively proposed to simulate reality and to simplify our calculations.**

P9

**Here are the assumptions of Model A.** In Model A, we would **consider the single-aisle case (although in our presentation we tend to combine it with other types of aircraft due to their similar deduction methods)**. It’s important to note that since we’ll vary the stowing time of passengers in the later slides, the first assumption is rather reasonable. **The time wasted while passengers try to stuff extra luggage into their seats is qualitatively equivalent to that spent while stowing those extra luggage.**

**For the second assumption on this slide, we disambiguated the expression in or essay by using this interpretation: passengers always maintain maximum theoretical speed. A corollary to this is that queuing has the same effect on a *block* of adjacent passengers.**

P10

**Here are the last two assumptions. We’ll explain the second assumption soon afterwards.**

P11

**As for the velocity, we assume that the velocity in a certain cell remains constant.** This means that V\_i (A) could be thought as points on the velocity function. Though it might not be so realistic, **this assumption enables us to simplify the calculation for the velocity (for it was only used when calculating distance) and wouldn't cause a lot of inaccuracy** (meaning that it wouldn't cause much change to the total time), as shown in the graph. This is partly because **the basic timestep is only 1/6 sec, a very short time** that wouldn't influence the velocity and distribution of passengers much, and thus according to the formula of calculating velocity, **the velocity in a certain timestep wouldn't change much**, so it can be seen as a constant.

P12

**In the model, the definition of time and velocity is different from SI, and this is the relationship between SI time and velocity and ours. We make these changes to make the calculations simpler.**

P37

**Disembarking is almost just the reverse of boarding,** for the motion is just the reverse from boarding. **So the best strategy should be similar to boarding.** However, for there’s no offering cells, the passengers have already been in an ideal queue, thus spending less time than boarding because of higher parallelity.

P38

Besides the total time, passengers’ **satisfaction** is also an important factor to take into account. In real-life experiences, dissatisfaction mainly comes from queuing and offering seats. And according to the strict sequence, some fellow passengers may be split, causing some dissatisfaction (though mistakenly not written in the essay). **The total dissatisfaction index is the weighted sum of the three factors.**

P39

**The weights of the factors are respectively 1, 250 and 10. The reason for 1 is for standardization, and 250 and 10 are according to real-life experience, and also to unite magnitudes to make the ultimate dissatisfaction index combine the three factors.**

P52

For the Flying Wing aircraft, as we've already divided it into four blocks, we define the intersection point of the main aisle and the ith block aisle as (i, 0, 0), and the ith block aisle as its x-grid. And for the TETA aircraft, we define the entrance cell in the left as (0, 0), and the direction of the two aisles as the x-grid. **The seats with a negative x-coordinate are the first class. And for the rest, passengers with seats x-coordinated 1 and 9 would board first, while those with 4 and 6 board last.**

P53

**Here is the coordinates for the TETA Aircraft.**

P55

**Here are the main ideas when we apply the model to different aircrafts.** TETA and the Flying Wing are two kinds of multi-aisle aircrafts, and we found that **they can be divided into smaller individual parts similar to ordinary one-aisle aircrafts.**

P56

To optimize the whole plan, it’s obvious that we need to **optimize the boarding sequence inside groups,** and then we need to **optimize the between-group sequences.**

P57

To ensure that every cell is used, we decide to **arrange a few inner group passengers to fill up empty blocks.**

P1

**Time and efficiency play a vital role in air transportation. For normal passenger flights, sections which require a great amount of time include the boarding and disembarking of passengers. Therefore, it's necessary to build a model which provides the best strategy for different types of planes and on various occasions.**

P2

**To begin with, we will introduce the overall boarding process, which is shown in this chart on the slide. While boarding a plane, passengers will first go to their assigned seat, put their luggage on the rack and then get seated. While a passenger is stowing their bags, other travelers who are stuck behind should wait until the passenger finishes the process, which will cause a queue.**

P13-P15

In the first model, we divide the variables into three types. **Constant A,** which is marked as A in this table refer to the constants which will not change in the whole scope. **Constant B** may change in the whole scope but won’t change when it comes to a certain set of passengers and plane types. **Variables** will for different initial sequences of passengers. (the slide change through the speech)

P58

We have concluded some of the strengths of our model.

**First, accuracy. In our model, we take several special situations into consideration. Also, we use several programs to facilitate our calculation**. This makes our result reasonable and precise.

**The second strength is universality**. In our model, **we succeeded in achieving visualization of the plane and successfully simulated the whole process of different boarding methods shown in the video clips just now.** This means that our model can be **applied to a variety of problems.**

**Finally, our model bears efficiency**. As shown in the second model, **we use a program to facilitate our calculations in finding the best strategy.** Therefore**, a lot of time is saved** and it proved that our model has efficiency.

P59

Also, we have found some weaknesses that needed to be improved.

**The first problem is complexity**. We introduce a great many variables and a variety of explanations in our model. **Some of them are a little bit abstract** and some of our calculations conducted by programs aren't shown in this essay. This will **make our model more complex and less easy to understand**.

**The second weakness is that our model is difficult to operate** when it comes to reality. As can be seen in the descriptions above, **our model provides a plan with some details that must be strictly obeyed**. **This will increase the difficulty for the crews to exert this plan. However, we have thought of a method that can ease this difficulty.** **When there’s a passenger ahead waiting, we can first let him get to his seat.** According to our sensitivity analysis, **this will not have a big impact on our boarding time**. Therefore, this kind of strategy is somehow reasonable and **flexible.**

P60

**Based on our model, we write a letter to provide the airline executives with some suggestions**. First, we **point out two important factors** in the whole boarding and disembarking process---- **hommization and efficiency**. **Secondly, we draw a simple chart to illustrate our plan**. **Finally, we offer some simple tips** that could be applied to all kinds of planes. **Airline executives need to prevent passengers from being stuck in general aisles, provide passengers with enough space to place their luggage and prevent queue-jumping.**